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Stress Analysis of Metallic Pressure Vessels with Circumferential Mismatch using Finite Element Method

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Abstract—Nonlinear finite element analysis (FEA) has been carried out on cylindrical and spherical pressure vessels having mismatch in the circumferential joint, with equal and unequal shell thickness/radius on either side of the joint. HSLA 15CDV6 is used as the vessel material, having application in aerospace structures. Stress factors are compared for cylindrical and spherical shell with different mismatch. Deformations and elastic stress distribution along meridional distance is presented graphically for different thickness ratio.

Index Terms—Circumferential joint, Finite element analysis, mismatch, nonlinear analysis, pressure vessel, stress factor

1 Introduction

Pressure vessels are widely used in the space, oil, chemical, nuclear power and many other industries. In the fabrication of pressure vessels various segments are joined together by welds or some other means to form a complete structure. In pressure vessels certain regions were exist, where continuity of the structure cannot be satisfied by the membrane forces alone. Such regions are known as discontinuity regions and the associated stress is discontinuity stress. Discontinuities in thickness, radius and slope have major role in the manufactured product. This causes additional bending stress in the discontinuity region and the stress distribution in the discontinuity region may change. It is desirable to reduce the number and magnitude of discontinuity to a minimum. In the context of effects of distortion at welded joints, the distortions considered are mismatch, weld sinkage (peaking or angular mismatch) and out of roundness. To predict the actual structural behaviour, it is necessary to go for geometric and material nonlinear analysis. The finite element analysis is done by utilizing the commercial software package ANSYS [12].

P. T Bizon [1] has examined the elastic stresses in a cylinder with mismatch and/or thickness change in the circumferential joint. A. Subhananda Rao et al. [2] have discussed on the effect of mismatch in the longitudinal joint of maraging steel motor cases. T. Aseer Brabin [3] et al. has obtained the elastic stress distribution at circumferential joint of cylindrical pressure vessel having misalignment using finite element analysis. R.H. Johns and T.W. Orange [4] have provided the theoretical stress distribution of shell type structures having discontinuity.

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1.1 Mismatch and Abrupt Thickness Change

This discontinuity obtained due to non coincidence of middle surface of the shell in the circumferential joint is shown in Figure (1). N_1 and N_2 are the membrane stress per unit length normal and parallel to the misalignment respectively. The primary aspect of the discontinuity is that a moment is introduced in the shell. A typical model of mismatched joint can be considered to analyze the effect of mismatch and/or thickness change as shown in Figure (2).

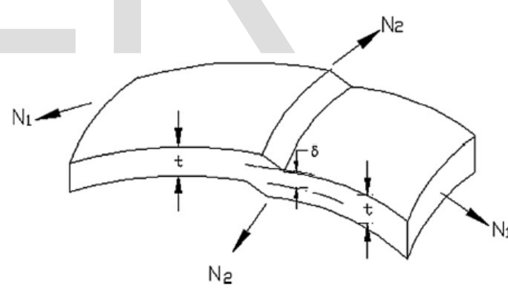

Figure1. Mismatched circumferential joint in cylindrical pressure vessel

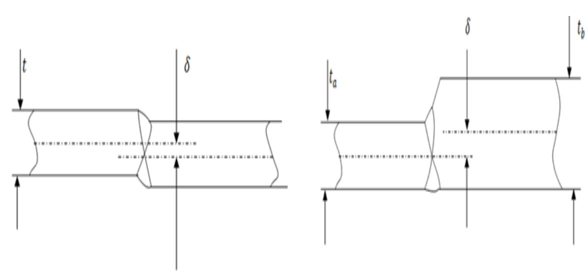

Figure2. Typical model of mismatched joint

The expression for additional bending stresses normal to mismatch ($\sigma_{L,b}$)is given below.

$$\sigma_{L,b} = \frac{3\delta}{t} * \sigma_{L,m} \qquad (1)$$

Here $\sigma_{L,m}$ is membrane stress normal to mismatch and δ is

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the offset of centre lines. Equation (1) and Equation (2) are applicable for shells with equal thickness. The expression for additional bending stresses parallel to mismatch ($\sigma_{H,b}$) is given as

$$\sigma_{H,b} = \frac{3\delta}{t} * \nu * \sigma_{H,m} \tag{2}$$

$\sigma_{H,m}$ is the membrane stress parallel to mismatch, ν is Poisson's ratio. The peak stress normal (σ_1) and parallel (σ_2) to the mismatch can be written as follows.

$$\sigma_1 = \sigma_{L,b} + \sigma_{L,m} \tag{3}$$

$$\sigma_2 = \sigma_{H,b} + \sigma_{H,m} \tag{4}$$

Mismatch can also be occurring in joining shells of different thickness. The maximum bending stress at the surface of thinner plate is given by the expression below [3].

$$\sigma_{L,b} = 3\sigma_{L,m}\left(\frac{t_b}{t_a} - 1\right)\left\{1 + \left(\frac{t_b}{t_a}\right)^3\right\}^{-1} \tag{5}$$

$$\sigma_{H,b} = \nu\,\sigma_{L,b} \tag{6}$$

The peak stress normal (σ_1) and parallel (σ_2) to the mismatch can be written as Equation (3) and (4). The stress magnification factors normal (K_1) and parallel (K_2) to mismatch are [3].

$$K_1 = \frac{\sigma_1}{\sigma_{L,m}} = 1 + \frac{\sigma_{L,b}}{\sigma_{L,m}} \tag{7}$$

$$K_2 = \frac{\sigma_2}{\sigma_{H,m}} = 1 + \frac{\sigma_{H,b}}{\sigma_{H,m}} = 1 + \nu\frac{\sigma_{L,b}}{\sigma_{H,m}} \tag{8}$$

The nonlinear analysis considers the thickness changes and mismatch of the joint. The geometry of the joint can be described by the thickness ratio (c) and mismatch factor (m) [8].

$$c = \frac{t_a}{t_b} \tag{9}$$

$$m = \frac{2\,(R_b - R_a)}{t_a + t_b} \tag{10}$$

The sign of m depends on whether (R_b - R_a) is positive or negative. In the influence coefficient approach, the nonlinear influence coefficients differ from linear influence coefficient by quantities that contain a pressure nonlinearity parameter ϱ. This parameter is a function of internal pressure (P) as well as geometry [8].

$$\rho = \frac{P}{\frac{2}{\sqrt{3(1-\upsilon^2)}} E \left(\frac{t}{R}\right)^2} \tag{11}$$

1.2 Stress Factors

In the plot between mismatch factor and stress factor, stress factor is used to relate the stress at particular point to an easily determined reference stress (here meridional membrane stress, $\sigma_{L,m}$). K_L and K_H are the stress factors corresponding to the meridional and hoop direction.

2 Finite Element Analysis

Non linear finite element analysis has been carried out on cylindrical and spherical pressure vessels having mismatch in the circumferential joint as shown in the Figure (3). The material used for the vessel is HSLA 15CDV6 having Young's modulus (E) 206010 MPa, Poisson's ratio of 0.3 and Yield strength 834 MPa. An axi-symmetric model with PLANE 42 element is used in the software package ANSYS. The geometric details of the vessels used for the comparison of stress magnification factors and for elastic stress distribution is given in Table 1, applied with internal pressure of 1.034 MPa [3]. The length of vessel is taken in such a way that the discontinuity stress diminishes and the membrane stress alone exists at the end away from the junction. Cylindrical pressure vessel with equal shell thickness is analyzed with 658 elements and 760 nodes and that of unequal shell thickness is analysed with 285 elements and 188 nodes. Spherical pressure vessel with unequal shell thickness is analyzed with 336 elements and 287 nodes.

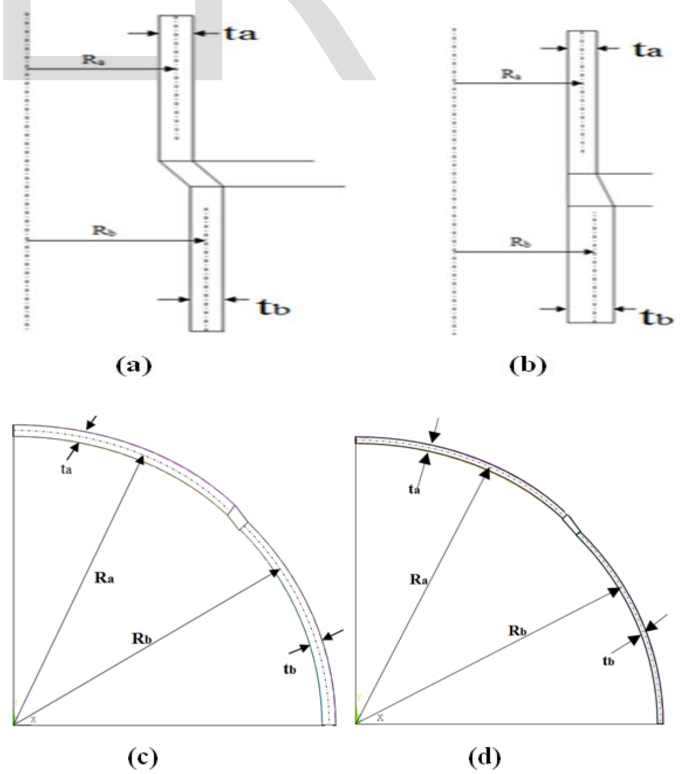

Figure3. Pressure vessels with mismatch in the circumferential joint (a) cylindrical with equal shell thickness (b) cylindrical with unequal shell

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thickness (c) spherical with equal shell thickness (c) spherical with equal shell thickness

Table 1
Geometric details of pressure vessels having mismatch

t_a/t_b	R_a	R_b	t_a	t_b	δ	ϱ	m
1	151.40	152.60	2.00	2.00	1.20	0.02	0.60
0.4	369.73	371.60	2.50	6.25	1.88	0.05	0.40

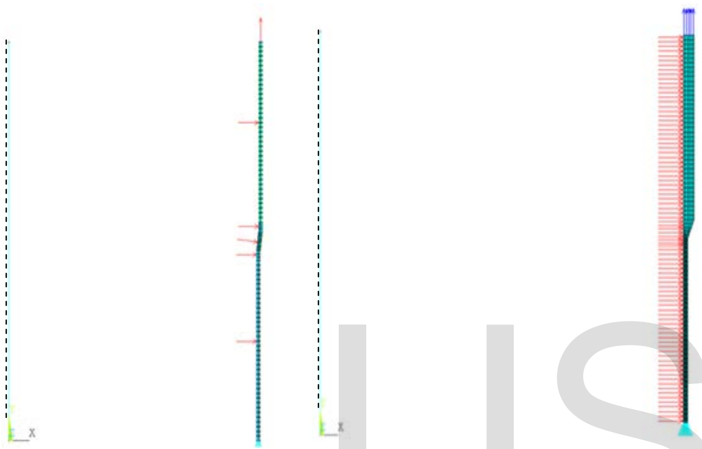

Figure4. Finite element model of cylindrical vessel with mismatched joint equal and unequal thickness

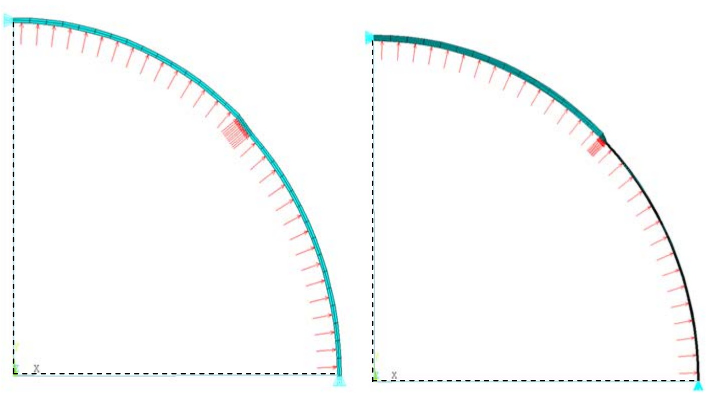

Figure5. Finite element model of spherical vessel with mismatched joint equal and unequal thickness

3 Results and Discussion

Stress distributions and deformations along meridional distance were plotted for different vessel geometry. The plots between mismatch factors and stress factor are given for both cylindrical and spherical vessels for the value of nonlinearity parameter (ϱ) is 1 for different thickness ratios.

3.1 Cylindrical Pressure Vessel

The stress magnification factors are evaluated for the two different thickness ratios and comparison is done between analytical and finite element solution is given in Table2. For the case of different shell thickness the peak stress and membrane stress values in the thin shell portion are considered for the comparison. Figure (6) and Figure (7) give the hoop, meridional and effective stress distributions and axial deformation of cylindrical pressure vessel of equal shell thickness along meridional distance. The discontinuity stresses are found to be an axi-symmetric distribution on either side of the joint. Similarly Figure (8), (9) and (10) shows the stress distribution and deformations for thickness ratio 0.4. The high stress levels experiences in the thinner shell region.

Table 2
Comparison of stress magnification factor for cylindrical pressure vessel

$\frac{t_a}{t_b}$	Stress magnification factor			
	Finite element analysis		Analytical solution	
			Equation(7)	Equation(8)
	K_1	K_2	K_1	K_2
1	2.10	1.26	2.8	1.27
0.4	1.39	1.03	1.27	1.04

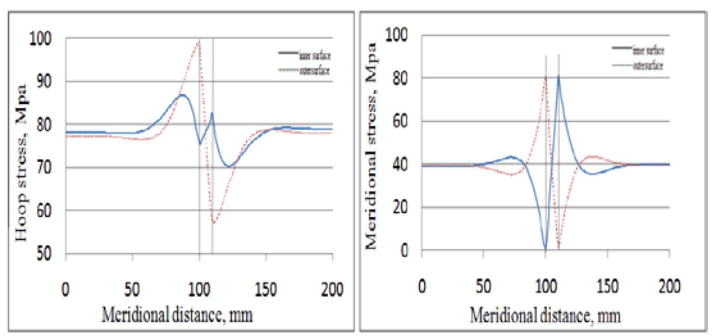

Figure 6. Hoop and meridional stress distribution for cylindrical vessel of equal shell thickness (ta/tb = 1)

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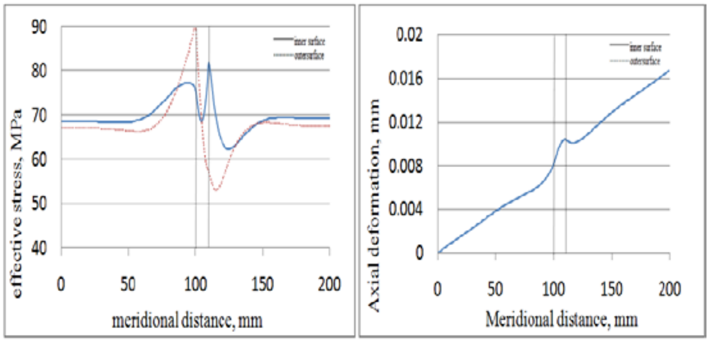

Figure 7. Effective stress distribution and axial deformation for cylindrical vessel of equal shell thickness (ta/tb = 1)

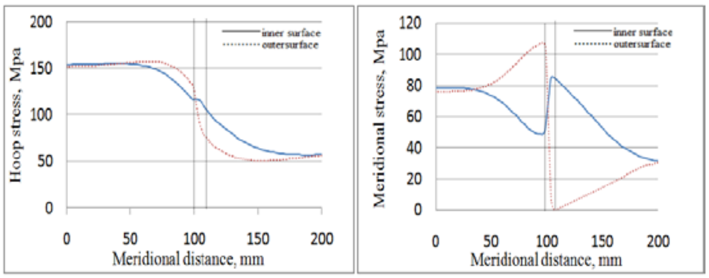

Figure 8. Hoop and meridional stress distribution for cylindrical vessel of unequal shell thickness (ta/tb = 0.4)

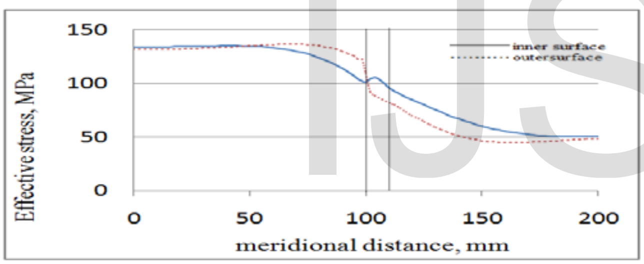

Figure 9. Effective stress distribution for cylindrical vessel of unequal shell thickness (ta/tb = 0.4)

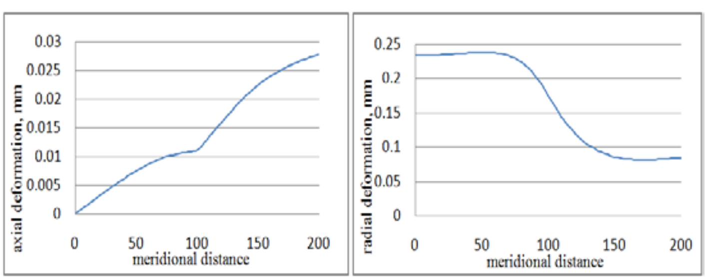

Figure 10. Axial and radial deformation for cylindrical vessel of unequal shell thickness (ta/tb = 0.4)

The stress factors at the junction region of two shells are plotted for the thickness ratio 1 and 0.4 is shown in Figure (11) and Figure (12). In the Figure (12) at the point of mismatch factor is zero, only the geometry change is considered as a discontinuity (thickness abruptly changes). The variation of mismatch factor is taken from -1 to +1.

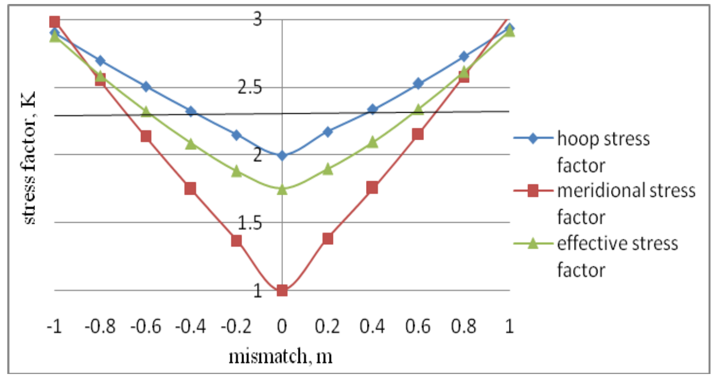

Figure 11. Stress factors at the junction region of two cylindrical shells (ta/tb = 1 and ρ=1)

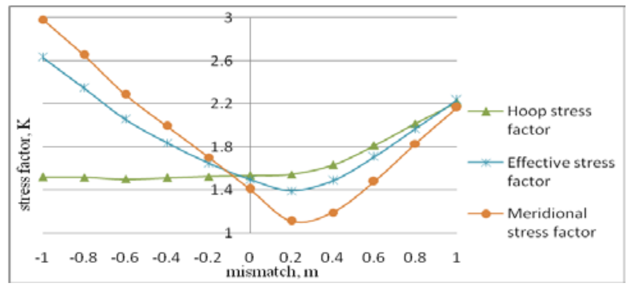

Figure 12. Stress factors at the junction region of two cylindrical shells (ta/tb = 0.4 and ρ=1)

3.2 Spherical Pressure Vessel

In the finite element model the mismatch in the spherical pressure vessel is provided at 45^0 angle. A comparison for the stress factors of spherical vessel is done by classical and finite element analysis is shown in the Table 3. Figure (13) and (14) give the hoop, meridional and effective stress distributions and axial deformation of cylindrical pressure vessel of unequal shell thickness along meridional distance. The high stress levels experiences in the thinner shell region.

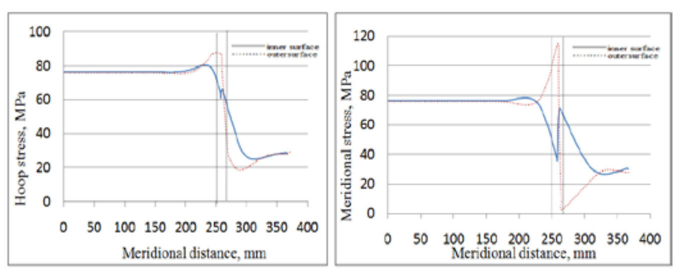

Figure 13. Hoop and meridional stress distribution for spherical vessel of unequal shell thickness (ta/tb = 0.4)

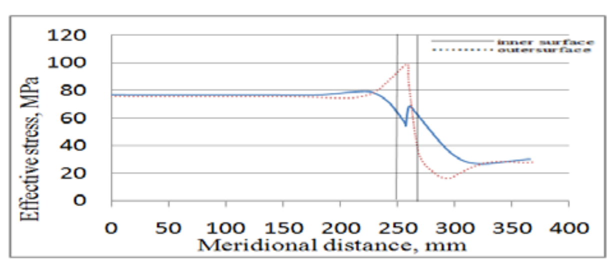

Figure 14. Effective stress distribution for spherical vessel of unequal shell thickness (ta/tb = 0.4)

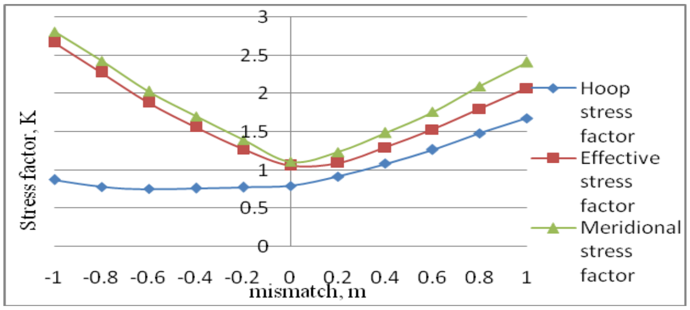

Figure 15. Stress factors at the junction region of two spherical shells (ta/tb = 0.4 and ρ=1)

3.2.1 Comparison of Stress Factors

By considering the similarity between the cylinder and sphere stress resultant and deformation, leads to the possibility of using the results for cylinders with mismatch for sphere. So that graph showing the mismatch verses stress factors for cylindrical pressure vessel should be applicable to spherical shell geometry. A conversion formula describes the same is given below. The stress factors of sphere with mismatch obtained from the finite element analysis (Figure 15) is compared with the stress factors obtained by the conversion formula. The conversion formula is [8],

$$(K_L)_{SR} = (K_L)_{CYL} - \left(1-\frac{\delta_{SR}}{\delta_{CYL}}\right)(K_{L0}-1) \tag{12}$$

$$(K_H)_{SR} = 1 + (K_H)_{CYL} - (K_{H0})\left(1-\frac{\delta_{SR}}{\delta_{CYL}}\right) - 2\frac{\delta_{SR}}{\delta_{CYL}} \tag{13}$$

Where, $$\frac{\delta SR}{\delta CYL} = \frac{N_{H_m} - \upsilon N_{L_m}}{PR_2\left(1-\frac{\upsilon}{2}\right)} \tag{14}$$

R_2 = Hoop radius, K_{H0}, K_{L0} = Intercepts (m= 0), N_{H_m} = Membrane hoop stress resultant
N_{L_m} =Membrane meridional stress resultant, SR = Shell of revolution, CYL = Cylinder
K_L = Meridional stress factor, K_H = Hoop stress factor

For the case of sphere N_{H_m} = N_{L_m} = PR/2, ν = 0.3, conversion formulae can be written as

$$(K_L)_{SR} = (K_L)_{CYL} - 0.588\,(K_{L0}-1) \tag{15}$$

$$(K_H)_{SR} = 1 + (K_H)_{CYL} - 0.588\,(K_{H0}) + .177 \tag{16}$$

These formulas are used to calculate stress factors for the sphere with ta/tb = 0.4 for varying mismatch and nonlinearity factors (ρ). The stress factors for the sphere with nonlinearity factor 1 (Much higher values than ρ = 1 is not experienced for metal pressure vessels) for various mismatch is compared with the finite element solution, are shown in the table below.

Table 3

Comparison of stress factors for spherical vessel (ta/tb = 0.4, ρ=1)

m	Meridional stress factor, K_L		Hoop stress factor, K_H	
	Equation (15)	FEA	Equation (16)	FEA
-0.4	1.61	1.56	0.82	0.76
-1.0	2.89	2.81	0.83	0.87
0.4	1.51	1.49	1.09	1.08
1	2.51	2.40	1.69	1.68

4 Conclusions

Nonlinear finite element analysis has been carried out on the cylindrical and spherical pressure vessels having mismatch in the circumferential joint utilizing the software ANSYS. The location of the peak stress value can only be obtained through FEA. The stress distribution along the meridional distance of spherical pressure vessel is similar to that of the trend in cylindrical pressure vessel. The results obtained from FEA shows good agreement with the results obtained from analytical method. The variation in results from FEA and analytical method may due to the consideration of the material and geometric nonlinearity in FEA.

5 Scope for Future Work

Future work can be directed to the distortion at circumferential or longitudinal welded joints as weld sinkage (angular distortion) and out of roundness, through finite element modeling.

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